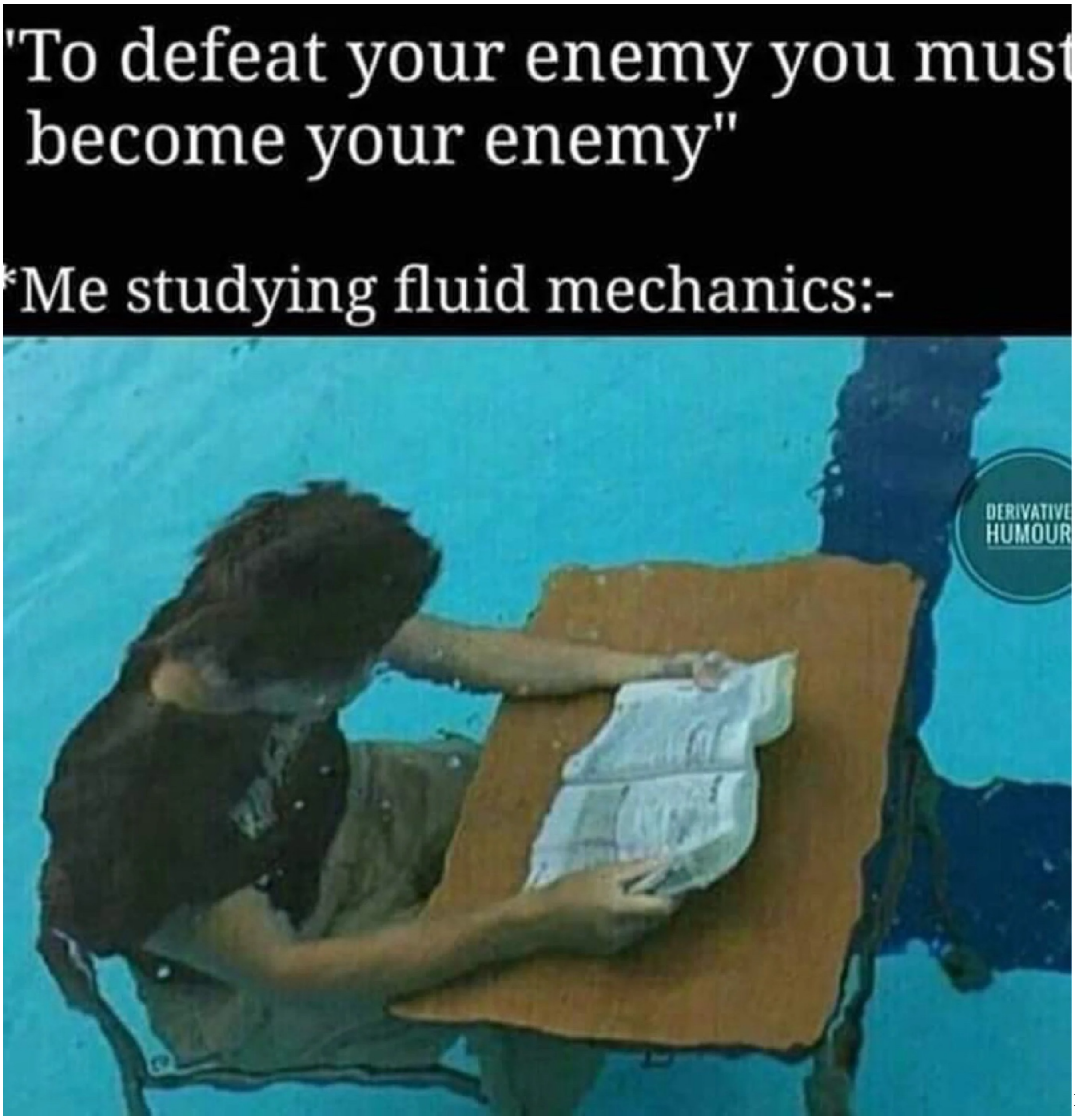
Project 1: Fluid Mechanics

Lev Gerasimov
Lg726
Downing college
Word count: 1997



[1]

[1] (Fluid Mech Meme, n.d.)

Table of Contents

1. Summary

The system of supplying water from one reservoir to other two contains series of two pipes. The first point of investigation is modeling the state values of the system as velocities of the water in two pipes while known in other. Next is to find the operational unit of the pump system and approximating the most efficient velocity of the water supply. The second iteration is to increase the accuracy of the system by increasing the complexity by considering the roughness of the pipes in calculations of the velocity and pressure changes. Third point of investigation is to model the requested ratio of outlet flows via changing the position of valve B with predetermined positions of valves A and C. The final aim of investigation is to discuss alternative pump options (single, in parallel, increase the rotational speed) and consider which of them is more advantageous be considering alternative flowrates.

2. Introduction

The chemical plant requires a water supply to support two-unit operations. The system of pipes is presented in figure 1.

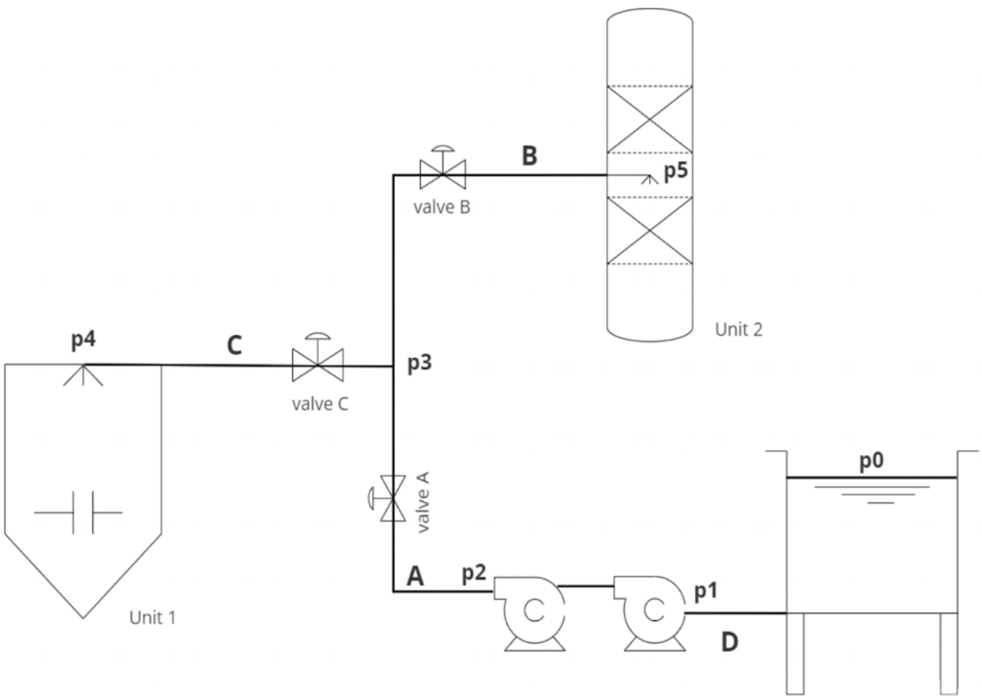


Figure 1 Pumps and Piping system

The water is held in the open reservoir with height $h_0 = 1.3\ m$. Water is piped to the system of two pumps in series through the pipe D with neglectable length at height $h_1 = 0\ m$. After the series of two identical pumps, water is pushed through the pump A of length $L_a = 18\ m$ and rises to height $h_3 = 5\ m$ where splits the stream into two: pipes B and C. Pipe C holds on the same height $h_3 = 5\ m$ and leads to the open reservoir via length $L_c = 23\ m$. Pipe B is rises to $h_5 = 10\ m$ to the open reservoir through the length $L_b = 26\ m$. All pipes due to imperfections have head losses equals to 1.

All pipes have incorporated valves (excluding D) which by rotating on specific angle increasing the head loss. Valves are open by default, unless stated opposite.

The main assumptions are that the water is incompressible fluid, hence, the continuity equation follows the form: $Q_1 A_1 = Q_2 A_2$.

The other assumption is that all pipes are rough and, unless stated otherwise, friction factor cannot be neglected in calculations.

The other assumption is used in calculations is that the point with pressure P_3 contains water at speed the same as the water in pipe A. Assumed that point of P_3 is the part of pipe A.

Another assumption is in the practical plane: all the systems of equations are solved by python program in google collab which can be found in the reference. The main library that was used is sympy. (Gerasimov, 2025)

3. Results

Problem 1

To determine the values of the pressures and velocities the system of four equations with four variables and one parameter can be used.

Beforehand, few assumptions needed to be stated:

a) The velocity of the liquid at point P_3 is considered as v_a.
b) The friction factors are constant, and the flow determined to be turbulent, hence equation 1 is used:

$$(\boldsymbol{eq\ 1})\ K = 4c_f\frac{L}{D}$$

where K is modified velocity head factor from the Darcy equation:

$$(\boldsymbol{Darcy\ eq})\ H_{fric} = 4c_f\frac{L}{D}\frac{v^2}{2g}$$

The system of equations 2-5 is obtained:

$$(\textbf{eq 2})\ h_2 + \frac{v_a^2}{2g} + \frac{P_2}{\rho g} = h_3 + \frac{v_a^2}{2g}\left(1 + K_a + 4c_f\frac{L_a}{D_a}\right) + \frac{P_3}{\rho g}$$

$$(\textbf{eq 3})\ h_3 + \frac{v_a^2}{2g} + \frac{P_3}{\rho g} = h_4 + \frac{v_c^2}{2g}\left(1 + K_b + 4c_f\frac{L_c}{D_c}\right) + \frac{P_4}{\rho g}$$

$$(\textbf{eq 4})\ h_3 + \frac{v_a^2}{2g} + \frac{P_3}{\rho g} = h_5 + \frac{v_b^2}{2g}\left(1 + K_c + 4c_f\frac{L_b}{D_b}\right) + \frac{P_5}{\rho g}$$

$$(\textbf{eq 5})\ v_a\frac{\pi d_a^2}{4} = v_b\frac{\pi d_b^2}{4} + v_c\frac{\pi d_c^2}{4}$$

Where equations 2-4 are Bernoulli equations of the system and equation 5 is continuity equation between pipes A and B&C.

The unknown values are v_b, v_c, P_3, P_4. The parameter that will be used to solve the system of equations 2-5 is v_a. c_f equals 0.005. The value for Q_a can be calculated by the equation 6:

$$(\boldsymbol{eq\ 6})\ Q_a = v_a\frac{\pi d_a^2}{4}$$

From the continuity equation between points D and A and Bernoulli equation between P_0 and P_1 can be derived the P_1. In Bernoulli equation the friction in pipe D is neglected. Due to the same diameter in pipes A and D, the velocity of liquid in pipe D assumed to be equal to v_a.

When Q_a is calculated, the value of the P_{pumps} can be calculated using the equation 7:

$$(\textbf{eq 7})\ P_{pumps} = 2 \cdot \Delta P = 2 \cdot (a - b \cdot Q_a^2)$$

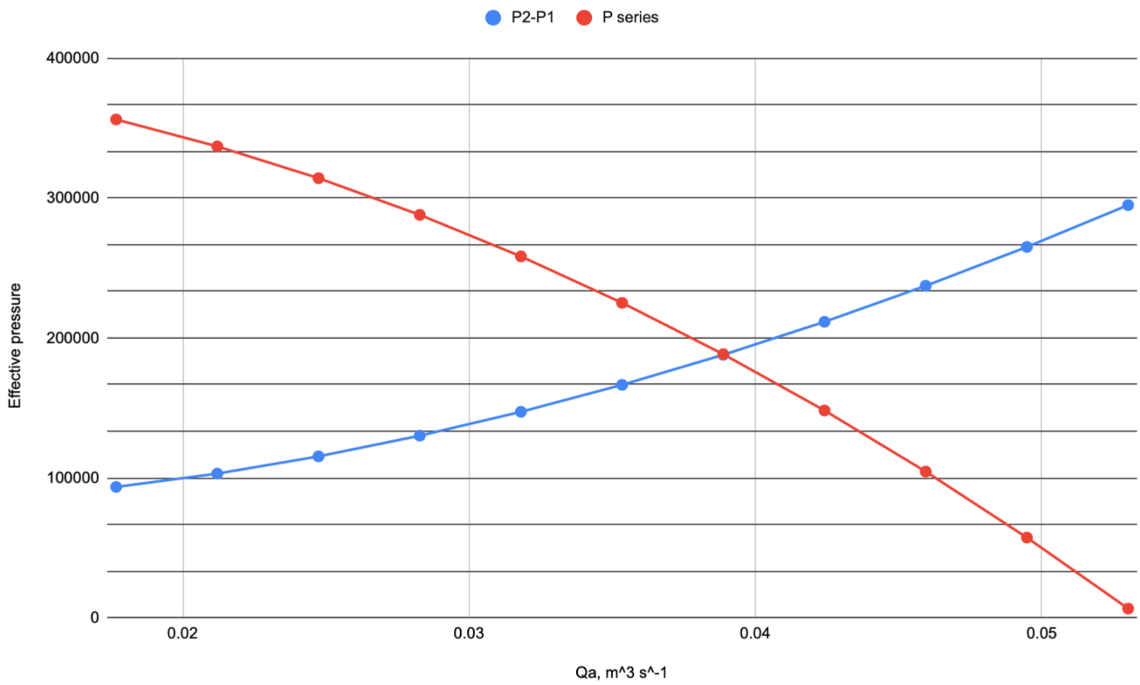
Table 1 for range of values v_a is obtained:

$v_a\ ,ms^{-1}$	$v_b\ ,ms^{-1}$	$v_c\ ,ms^{-1}$	$P_1\ ,Pa$	$P_2\ ,Pa$	$P_3\ ,Pa$	$P_2 - P_1, Pa$	P_{pumps}, Pa	Q_a, m^3s^{-1}
1.00	1.18	3.42	11753	105235	54485	93482	356281	0.018
1.20	1.84	3.67	11313	114322	62824	103009	337044	0.021
1.40	2.45	3.98	10793	126126	73744	115333	314310	0.025
1.60	3.02	4.32	10193	140313	86911	130120	288079	0.028
1.80	3.57	4.69	9513	156721	102163	147208	258350	0.032
2.00	4.11	5.07	8753	175264	119414	166511	225123	0.035
2.20	4.63	5.47	7913	195893	138615	187980	188399	0.039
2.40	5.15	5.87	6993	218577	159735	211584	148177	0.042
2.60	5.66	6.28	5993	243297	182755	237304	104458	0.046
2.80	6.16	6.70	4913	270042	207663	265129	57241	0.049
3.00	6.66	7.12	3753	298801	234450	295048	6527	0.053

Table 1 Values of Pipe System

Problem 2

From the table above the graph of $P_2 - P_1$ and P_{pumps} over Q_a can be plotted. The intersection of these lines is the maximum efficiency of the pump.



Graph 1 Pump Characteristics over Q_a

From the graph can be seen that the intersection appears at Q_a equals $0.039\ m^3 s^{-1}$. Therefore, from the value of Q_a the value of the velocity, Reynolds number and c_f of A, B, C can be calculated. The roughness of commercial pipes considered to be $\varepsilon = 0.046\ mm$.

The Reynolds number for each pipe is around $3.5 \cdot 10^5$, which is a turbulent flow. Hence the roughness of the pipe is non-zero value, the equation for the friction factor is the equation 8:

$$(\boldsymbol{eq\ 8})\ c_f = 1.375 \cdot 10^{-3} \left[1 + \left(2 \times 10^4 \frac{\varepsilon}{D} + \frac{10^6}{Re} \right)^{\frac{1}{3}} \right]$$

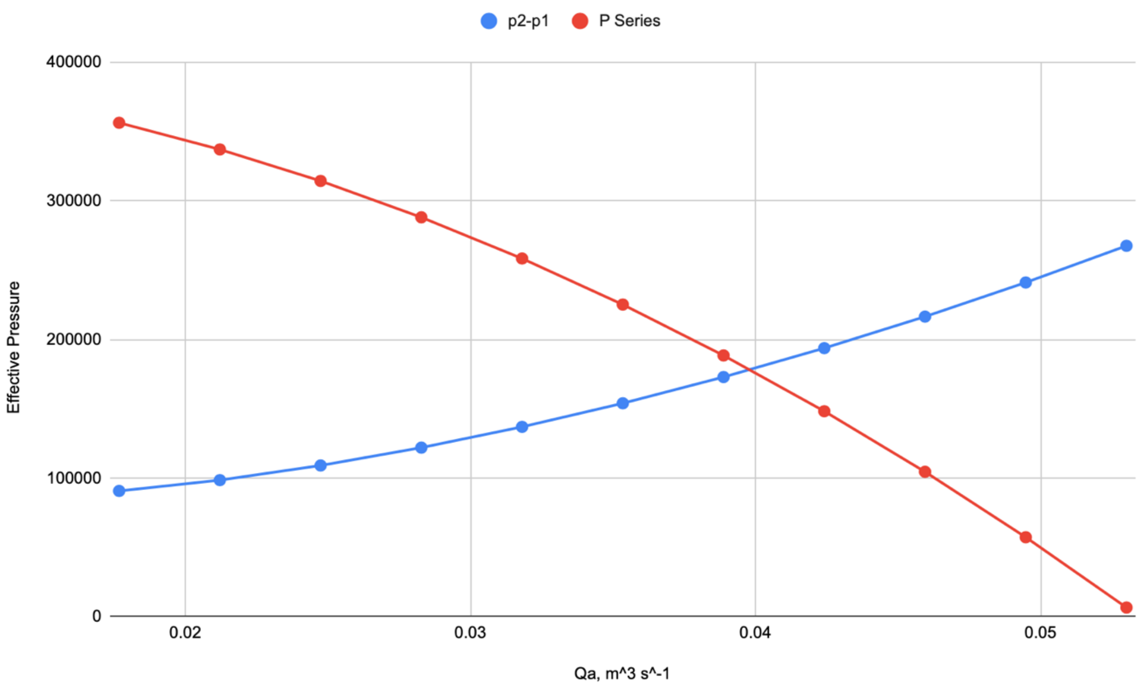
	Pipe A	Pipe B	Pipe C
$Velocity, ms^{-1}$	2.20	4.33	5.77
$Volume\ flowrate, m^3 s^{-1}$	0.039	0.018	0.021
Re	$3.3 \cdot 10^5$	$3.2 \cdot 10^5$	$3.8 \cdot 10^5$
c_f	0.0042	0.0043	0.0042

Table 2 System with Roughness

The friction factors appear to be 20% smaller than one was used in calculations in question 1. Therefore, to recalculate, velocities in Pipes A, B, C the new c_f values going to be substituted into the four equations presented in part 1. The Reynolds number is assumed constant, or its changes will not affect c_f values.

v_a, ms^{-1}	v_b, ms^{-1}	v_c, ms^{-1}	P_1, Pa	P_2, Pa	P_3, Pa	$P_2 - P_1, Pa$	Q_a, m^3s^{-1}	P_{pumps}, Pa
1.00	0.89	3.70	51837	102395	11753	90642	0.018	356281
1.20	1.57	3.94	58542	109764	11313	98451	0.021	337044
1.40	2.19	4.24	67802	119807	10793	109014	0.025	314310
1.60	2.76	4.59	79178	132088	10193	121895	0.028	288079
1.80	3.31	4.96	92465	146401	9513	136888	0.032	258350
2.00	3.83	5.35	107558	162640	8753	153887	0.035	225123
2.20	4.33	5.76	124396	180745	7913	172832	0.039	188399
2.40	4.85	6.17	142944	200680	6993	193687	0.042	148177
2.60	5.34	6.60	163178	222423	5993	216430	0.046	104458
2.80	5.83	7.03	185085	245958	4913	241045	0.049	57241
3.00	6.31	7.46	208654	271276	3753	267523	0.053	6527

Table 3 Value of System with Roughness



Graph 2 Pump characteristics over Q_a with friction

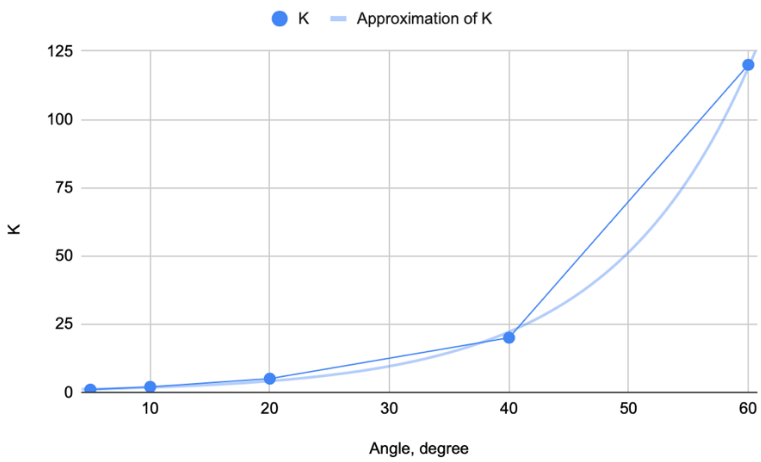
From the values in table 3 inner pressure has decreased. The velocity in pipe B has dropped while velocity in pipe C has risen. The operational point of the pump system has increased to the $0.040\ m^3 s^{-1}$, which is obtained from graph 2. The calculation approximation was the overestimation; hence, the exact calculations are more efficiently to calculate results more accurately. However, the simpler approximation was enough precise to estimate value with potential losses.

Problem 3

For the following problem the c_f value is considered from the problem 2 of the report. The value of the velocity of the liquid is considered as 2.2 ms^{-1} as the closest to the operational point value.

The values of the K were compared with the Perry's handbook (Green and Southard, 2019, pp.6–18). The result is close to the butterfly valve, which behaves close to the exponential.

Hence, the approximation of the values of K to the different angles are going to follow value of exponential function.



Graph 3 K over the Angle with approximation

Using the approximation function (also known as the trendline with Pearson coefficient 0.999 which makes approximation valuable) $K = 0.796 \cdot e^{0.084 \cdot \alpha}$, where α is the angle of the valve.

The table 4 is obtained:

Angle degrees	K approximated	K exact
5.00	1.17	1.00
10.00	1.78	2.00
20.00	4.13	5.00
30.00	9.56	Unknown
40.00	22.14	20.00
45.00	33.69	Unknown
60.00	118.79	120.00

Table 4 Approximation of the head loss

The value of approximation is showed up to 2 d.p. to declare the accuracy of the approximation. In calculations the value is rounded up to 0 d.p. due to the given value up to 0 d.p.

The table 5 of the results was obtained in requirement to solve system of equation 9-14:

$$(\textbf{eq 9})\ h_2 + \frac{v_a^2}{2g} + \frac{P_2}{\rho g} = h_3 + \frac{v_a^2}{2g}\left(2 + K_a + 4c_f\frac{L_a}{D_a}\right) + \frac{P_3}{\rho g}$$

$$(\textbf{eq 10})\ h_3 + \frac{v_a^2}{2g} + \frac{P_3}{\rho g} = h_4 + \frac{v_c^2}{2g}\left(2 + K_b + 4c_f\frac{L_c}{D_c}\right) + \frac{P_4}{\rho g}$$

$$(\textbf{eq 11})\ h_3 + \frac{v_a^2}{2g} + \frac{P_3}{\rho g} = h_5 + \frac{v_b^2}{2g}\left(2 + K_c + 4c_f\frac{L_b}{D_b}\right) + \frac{P_5}{\rho g}$$

$$(\textbf{eq 12})\ v_a\frac{\pi d_a^2}{4} = v_b\frac{\pi d_b^2}{4} + v_c\frac{\pi d_c^2}{4}$$

$$(\textbf{eq 13})\ \mathrm{Q_b} = 0.75\mathrm{Q_c}$$

v_a , ms^{-1}	v_b , ms^{-1}	v_c , ms^{-1}	K_a	Angle of A	K_c	Angle of C	K_b	Angle of B
2.20	4.33	5.77	0	0	0	0	0	0
2.20	4.33	5.77	34	45	34	45	60	52
2.20	4.33	5.77	34	45	10	30	17	37

Table 5 System values with closed valves

Part i

The position of the valve B with fully opened valves A and C is also a fully open position. The stream was already distributed at required ratio without additional closure of the valve B.

Part ii

The valve position of the B is determined to be at *52* degrees when both A and C are opened at *45* degrees.

Part iii

The valve position of the B is determined to be at *37* degrees when A is *45* degrees and C is *30* degrees opened

Problem 4

Reference point of calculation was the calculation of two pumps in series. In series the value of the pressure is accumulated (equation 14).

$$(\boldsymbol{eq}\ \mathbf{14})\ \Delta P = 2 \cdot (a - bQ^2)$$

Part i

The difference in calculation between single pump and two identical pumps in series is that the pressure of single pump cannot be accumulated twice. Therefore equation 15 is obtained:

$$(\mathbf{eq}\ \mathbf{15})\ \Delta P = a - \ bQ^2$$

Part ii

The difference between having pipes in series and pumps in parallel is that through the parallel only the volumetric flow is accumulated, while in series the head is accumulated instead. Therefore equation 16 is obtained:

$$(\mathbf{eq}\ \mathbf{16})\ \Delta P = a - \ b\left(\frac{Q}{2}\right)^2$$

Part iii

The proportionality equation can be considered as the main source of correlation between pressure difference and impeller rotational speed (equation 17:

$$(\mathbf{eq}\ \mathbf{17})\ \frac{\Delta P}{\rho\omega^2 D^2} = \alpha - \beta\left(\frac{Q}{\omega D^3}\right)^2$$

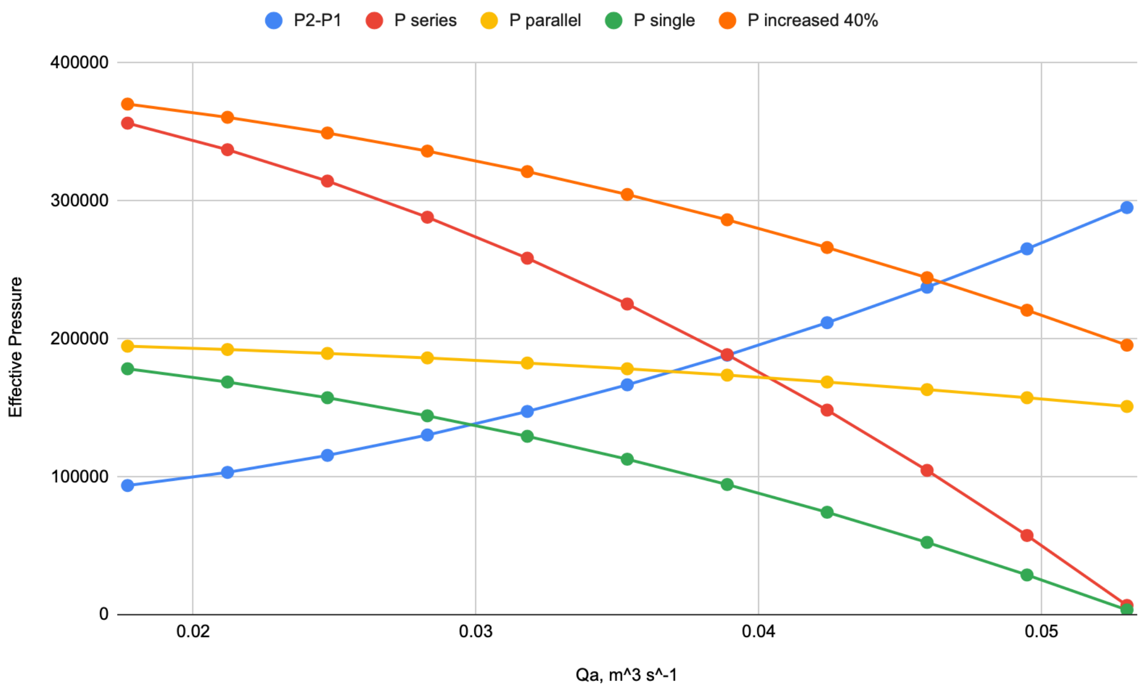
The rotational speed is proportional to the frequency of the impeller. Density and the impeller diameter are constant. From the equation noticeable that the pressure difference and β are proportional to the ω^{-2}; otherwise α is proportional to ω^2, hence, increasing the frequency, the free coefficient will increase quadratically proportional to the frequency. Therefore equation 18 can be obtained:

$$(\mathbf{eq}\ \mathbf{18})\ \Delta P \propto \alpha\omega^2 - \ bQ^2$$

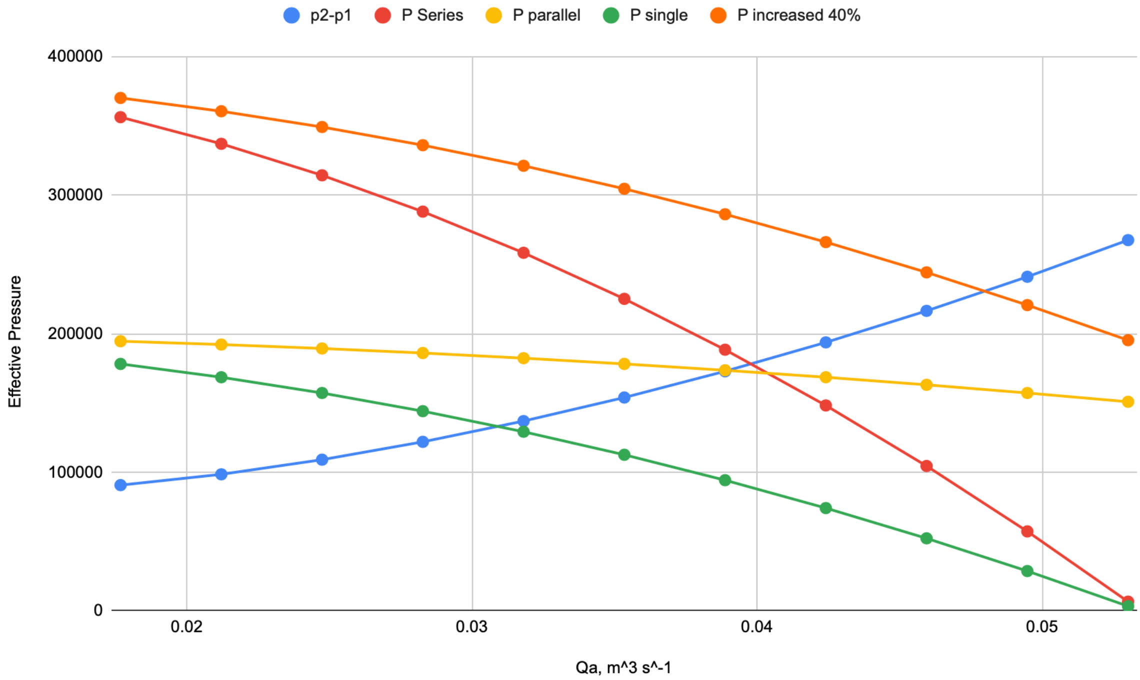
Or:

$$(\mathbf{eq}\ \mathbf{18})\ \mathbf{a}' = \mathrm{a}\left(\frac{\mathrm{w}'}{\mathrm{w}}\right)^2 ; \mathrm{b}' = \mathrm{b}$$

The graphs 4 and 5 are obtained when every pressure difference plotted against volumetric flowrate. (Numerical data in Appendix tables 7, 6 respectively to graphs). Graph 4 is plotted for constant $c_f = 0.005$. Graph 5 is obtained for real values of c_f.



Graph 4 Graph of the Power with $c_f = 0.005$



Graph 5 Graph of Power with real c_f

The difference between operational points of pumps in parallel and in series not that different; but if the pressure difference between P_1 and P_2 will be increased, the efficiency head of the parallel system is going to be higher, while shift to the left side of the graph makes pumps in series preferable choice. The shape difference between the single pumps does not appear, because the only change is the free coefficient which was roughly doubled in increased frequency pump. The main reason to choose either pump is the exactness of the operational point intersection. Single pump — for the lowest, single accelerated — for the highest.

Also, the difference can be considered between the graphs plotted over the Q_a with consideration of the roughness of pipes. The actual roughness shows that the operational point is systematically larger in terms of the volumetric flowrate.

4. Conclusion

The operational point of the simplified model with the constant friction factor $c_f = 0.005$ lies about $Q_a = 0.039\ m^3s^{-1}$ ($v_a = 2.2\ ms^{-1}$) through pipe A and series of pumps. This gives the maximum efficiency for the pump.

The correction of the model by considering roughness of the pipe changes the operational point of the system to the slightly larger values of Q_a ($0.040\ m^3s^{-1}$). The simple approximation found to be overestimation to prevent underestimation of resources.

The modeling of the fixed volumetric ratio in pipes B and C by adjusting the valve B with fixed valves A and C gave results in range between 0 and 52 degrees. With the opened valves the streamline is already distributed in a required ratio, hence the angle not needed to be changed. While for the other values of valves A and C the valve B needed to be adjusted moderately up to other valves.

The difference in the configurations have seemed to be significant in changing for the changing pressure difference between P_1 and P_2. For increasing the difference, the single pump with increased impeller speed is the most efficient choice. The difference between series and parallel configuration is that the operational unit is approximately the same for the parallel configuration, slightly depending on the volumetric flowrate. On the other hand, the operational head of the system in series plummets significantly as the volumetric flowrate increases.

5. References

Fluid Mech Meme. (n.d.). Available at: https://share.google/images/6PCg2BehlYYowF5Ro.

Green, D.W. and Southard, M.Z. (2019). Perry's chemical engineers' handbook. 9th ed. New York, Ny: Mcgraw-Hill Education, pp.6–18.

Gerasimov, L. (2025). Solutions for the Fluid Project Gerasimov. [online] Google.com. Available at: https://colab.research.google.com/drive/1ZY5_oaSbgLOWqMTj8fPF-QZ19f-PYJ5n [Accessed 29 Oct. 2025].

6. *Appendix*

Va	Vb	Vc	P3	P2	p1	p2-p1	P Series	Qa	P parallel	P single	P increased 40%
1.00	0.89	3.70	51837	102395	11753	90642	356281	0.018	194535	178140	370140
1.20	1.57	3.94	58542	109764	11313	98451	337044	0.021	192131	168522	360522
1.40	2.19	4.24	67802	119807	10793	109014	314310	0.025	189289	157155	349155
1.60	2.76	4.59	79178	132088	10193	121895	288079	0.028	186010	144039	336039
1.80	3.31	4.96	92465	146401	9513	136888	258350	0.032	182294	129175	321175
2.00	3.83	5.35	107558	162640	8753	153887	225123	0.035	178140	112561	304561
2.20	4.34	5.76	124396	180745	7913	172832	188399	0.039	173550	94199	286199
2.40	4.85	6.17	142944	200680	6993	193687	148177	0.042	168522	74089	266089
2.60	5.34	6.60	163178	222423	5993	216430	104458	0.046	163057	52229	244229
2.80	5.83	7.03	185085	245958	4913	241045	57241	0.049	157155	28620	220620
3.00	6.31	7.46	208654	271276	3753	267523	6527	0.053	150816	3263	195263

Graph 6 Data for Pumps with real c_f

Va	Vb	Vc	p1	p2	p3	P2-P1	P series	Qa	P parallel	P single	P increased 40%
1.00	1.18	3.42	11753	105235	54485	93482	356281	0.018	194535	178140	370140
1.20	1.84	3.67	11313	114322	62824	103009	337044	0.021	192131	168522	360522
1.40	2.45	3.98	10793	126126	73744	115333	314310	0.025	189289	157155	349155
1.60	3.02	4.32	10193	140313	86911	130120	288079	0.028	186010	144039	336039
1.80	3.57	4.69	9513	156721	102163	147208	258350	0.032	182294	129175	321175
2.00	4.11	5.07	8753	175264	119414	166511	225123	0.035	178140	112561	304561
2.20	4.63	5.47	7913	195893	138615	187980	188399	0.039	173550	94199	286199
2.40	5.15	5.87	6993	218577	159735	211584	148177	0.042	168522	74089	266089
2.60	5.66	6.28	5993	243297	182755	237304	104458	0.046	163057	52229	244229
2.80	6.16	6.70	4913	270042	207663	265129	57241	0.049	157155	28620	220620
3.00	6.66	7.12	3753	298801	234450	295048	6527	0.053	150816	3263	195263

Graph 7 Data for the Pumps with $c_f = 0.005$